Controlling spatiotemporal chaos via phase space compression

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We present a simple and effective method for controlling spatiotemporal chaos (STC) via phase space compression, by compressing the evolution orbit of the chaotic attractor. In numerical simulations, we obtain global and local control in coupled map lattice (CML) systems by the same phase space compression in different situations, and find that the functional relationship of control results to control parameters in a certain region is the same as the local dynamics expression of the CML. According to the control equation, using different phase space compressions we successfully control a CML exhibiting STC into various desired stable states.

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I. INTRODUCTION

Controlling spatiotemporal chaos (STC) has been given much attention by scientists and technologists in recent years [1–19] because developments in controlling STC offer opportunities for potentially practical applications. Generally speaking, there are two kinds of method for controlling STC, feedback control and nonfeedback control, and each one has both advantages and disadvantages. STC systems are commonly described by coupled map lattice (CML) models, which are relatively simple. However, only a few methods have been proposed for controlling such systems [1–9]. Feedback pinning [1–8] and constant pinning [9] have been used to control or suppress chaos modeled by CMLs.

In this paper, we will give an example of controlling STC in CMLs by phase space compression, which compresses the phase space orbit of the chaotic attractor. Similar methods are used to control temporal chaos [20,21]; to our knowledge, the method has not been used to control STC. Because nonfeedback control does not need any prior knowledge of the system or explicit changes of system parameters, it is easy to implement, and may be particularly convenient for experimentalists. When the system is under control, the controlling input does not vanish and the controlled target state may or may not be an unstable periodic orbit of the chaotic attractor. In numerical simulations, we first analyze the control results in the logistic map by using this method. Then we obtain global and local control in CMLs exhibiting STC by the same phase space compression in different situations, and find the functional relationship between control results and control parameters in a certain region. According to the control equation, using different phase space compressions we successfully control a CML exhibiting STC into various desired stable states.

II. CONTROL METHOD AND SIMULATION RESULTS

The model we used in this paper is a one-dimensional CML model, originally introduced by Kaneko [22]:

$$x_{n+1}(i)=(1-\varepsilon)f[x_n(i)]+\frac{\varepsilon}{2}\{f[x_n(i-1)]+f[x_n(i+1)]\}, \tag{1}$$

where $n=1,2,\ldots,N$ are the discrete time steps, $i=1,2,\ldots,L$ are the lattice sites, ε is the coupling strength to the nearest neighbor sites, periodic boundary conditions $x_n(i+L)=x_n(i)$ are imposed, and $f(x)$ governs the local dynamics. We choose $f(x)=\alpha x(1-x)$. With $L=1$, model (1) reduces to the well-known logistic map; the dynamics of this map might be either periodic or chaotic, depending on the nonlinear parameter α. $3.569\,945\,6\ldots<\alpha\leqslant 4$ is the chaotic region; $x^*=1-1/\alpha$ is an unstable steady state in the chaotic region.

Now, we consider the logistic map with phase space compression and take $\alpha=4$,

$$x_n=\begin{cases} x_n, & x_{\min}<x_n<x_{\max} \\ x_{\max}, & x_n\geqslant x_{\max} \\ x_{\min}, & x_n\leqslant x_{\min}, \end{cases} \tag{2}$$

$$x_{n+1}=4x_n(1-x_n),$$

where $x_{\max}, x_{\min}\in[0,1]$. Phase space compression is used to limit x_n only before iteration so our method is different from the methods in [20,21]. We use it to control chaos in the logistic map and give the control parameter ranges that lead to orbits of any desired period. To illustrate this, we consider only $x_{\min}=0$. We graphically analyze the iterative results of Eq. (2) with $x_{\max}=0.9$ in Fig. 1(a) and $x_{\max}=0.75$ in Fig. 1(b). The period-2 orbit and the unstable steady state x^* are obtained, respectively, and period-1 orbits will always be found when $x_{\max}\leqslant 0.75$. We now analyze the control method from another angle assuming $x_{n+1}=4x_{\max}(1-x_{\max})$ to be used instead of the logistic map in the interval $[x_{\max},1]$. Then the curve of the logistic map in this region changes into the dashed horizontal line shown in Fig. 1(a) and Fig. 1(b). Since the slope of the line is 0, orbits in this region are guaranteed to change from unstable to stable and each orbit is equivalent to the one at $x_{\max}$. In Fig. 1(c) we iterate the logistic map with $\alpha=4$ for all times, except $n=700$ to $n=1300$. During this control time, we iterate Eq. (2) with $x_{\max}=0.967$ and obtain a stable period-3 orbit. A bifurcation diagram that gives the values of successive iterates as a function of $x_{\max}$ in Eq. (2) is shown in Fig. 1(d). So, by choosing the parameters $x_{\max}$ and $x_{\min}$ appropriately in Eq. (2), one can stabilize any

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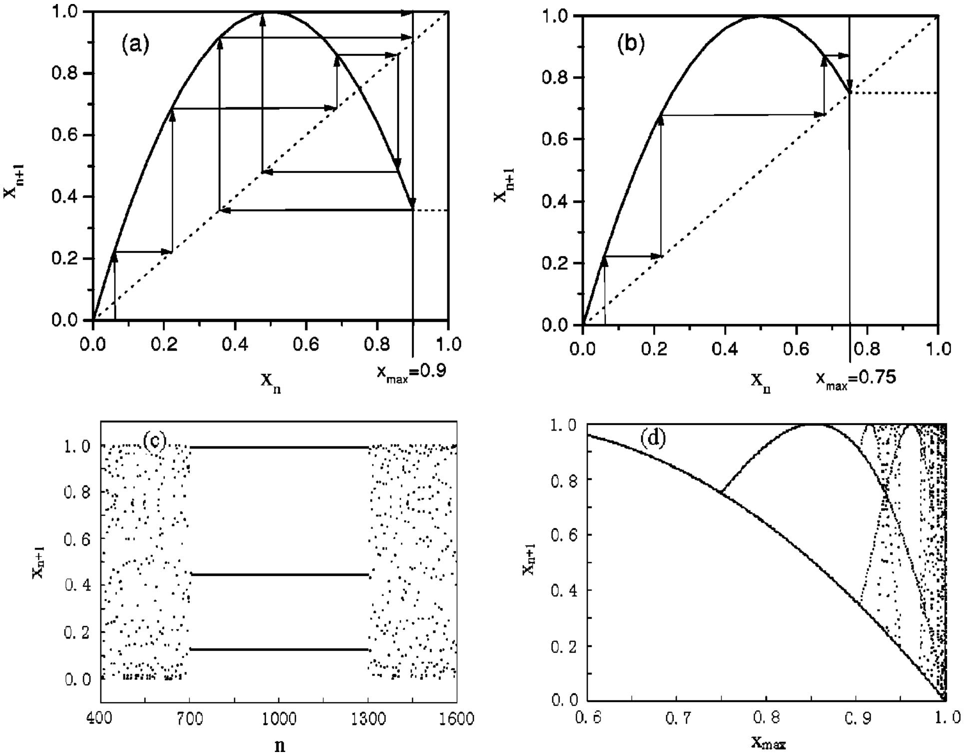

FIG. 1. (a) Plot of the iterate of Eq. (2) with $x_{\max}=0.9$, $x_{\min}=0$. The period-2 orbit is obtained. (b) Plot of the iterate of Eq. (2) with $x_{\max}=0.75$, $x_{\min}=0$. x^* is obtained. (c) Chaos control using phase space compression. Each point gives a value of x_{n+1}. The logistic map with $\alpha=4$ is used except during control from $n=700$ to $n=1300$. During control, Eq. (2) is used for the iteration with $x_{\max}=0.967$, $x_{\min}=0$. (d) A bifurcation diagram for Eq. (2) with $x_{\min}=0$. For each value of $x_{\max}$, 200 points are plotted following a transient of 100 iterations.

desired periodic orbit in the logistic map. In particular, the control result of period-1 obeys Eq. (6) below when $0 \leqslant x_{\max} \leqslant 0.75$.

Next, we select the motion of CML (1) that is fully developed turbulence at $\varepsilon=0.8, \alpha=4$ [1,2,5], and $L=64$. After transient iterations, the evolution orbits of the system represent a chaotic attractor in phase space; this chaotic attractor is limited in a bounded phase space V and its values are distributed in the interval [0,1]. To start to control STC in system (1) at the $(n+1)$th iterative, we select a subspace W, $W \subset V$, $W \not\supset \Phi$, and compress the orbits of the lattice $x_n(i), i=1,2,\ldots,L$, into W. Thus, $x_n(i)$ is changed to

$$x_n(i)=\begin{cases} x_n(i), & x_n(i)_{\min}<x_n(i)<x_n(i)_{\max} \\ x_n(i)_{\max}, & x_n(i) \geqslant x_n(i)_{\max} \\ x_n(i)_{\min}, & x_n(i) \leqslant x_n(i)_{\min} \end{cases} \tag{3}$$

$$i=1,2,\ldots,L,$$

where $x_n(i)_{\max} \in W$, $x_n(i)_{\min} \in W$ are the boundaries of subspace W. For global control with the same phase space compression

$$x_n(i)_{\max}=x_{\max},$$

$$x_n(i)_{\min}=x_{\min}, \tag{4}$$

$$i=1,2,\ldots,L, \quad x_{\max}, x_{\min} \in W.$$

For local control

$$x_n(i)_{\max}=\begin{cases} x_{\max}, & i=i_s \\ 1, & i \neq i_s, \end{cases}$$

$$x_n(i)_{\min}=\begin{cases} x_{\min}, & i=i_s \\ 0, & i \neq i_s, \end{cases} \tag{5}$$

$$i=1,2,\ldots,L, \quad x_{\max}, x_{\min} \in W,$$

where i_s is the selected control site. If $i=i_s$, $x_n(i) \in W$, the phase space orbit of the ith lattice is compressed into W; if $i \neq i_s$, $x_n(i) \in V$, the phase space orbit of the lattice remains unchanged.

A. Global control of STC with the same phase space compression

1. Homogeneous lattice

We show the effects of the same phase space compression on the CML (1) for a homogeneous lattice (same α and same ε at all sites). The results presented in Figs. 2(a)–2(c) show the space-time evolution of the CML (1) before and after control with $\varepsilon=0.8, \alpha=4$, $L=64$, and the initial condition of pseudorandom numbers uniformly distributed in the interval [0,1]. Every eighth time step is plotted after 10 000 iterations of transients; thus the time is $n/8-1250$ from 0 to 600. At time 200, phase space compression begins and phase space V changes into subspace W. From now on, we will always use such conditions unless specified otherwise. One can see that the motion of the system is fully turbulent from time 0 to 200 and the turbulent motion of STC can be successfully suppressed after the phase space compression is input. In Fig. 2(a), we fix the lower boundary of V unchanged at $x_{\min}=0$

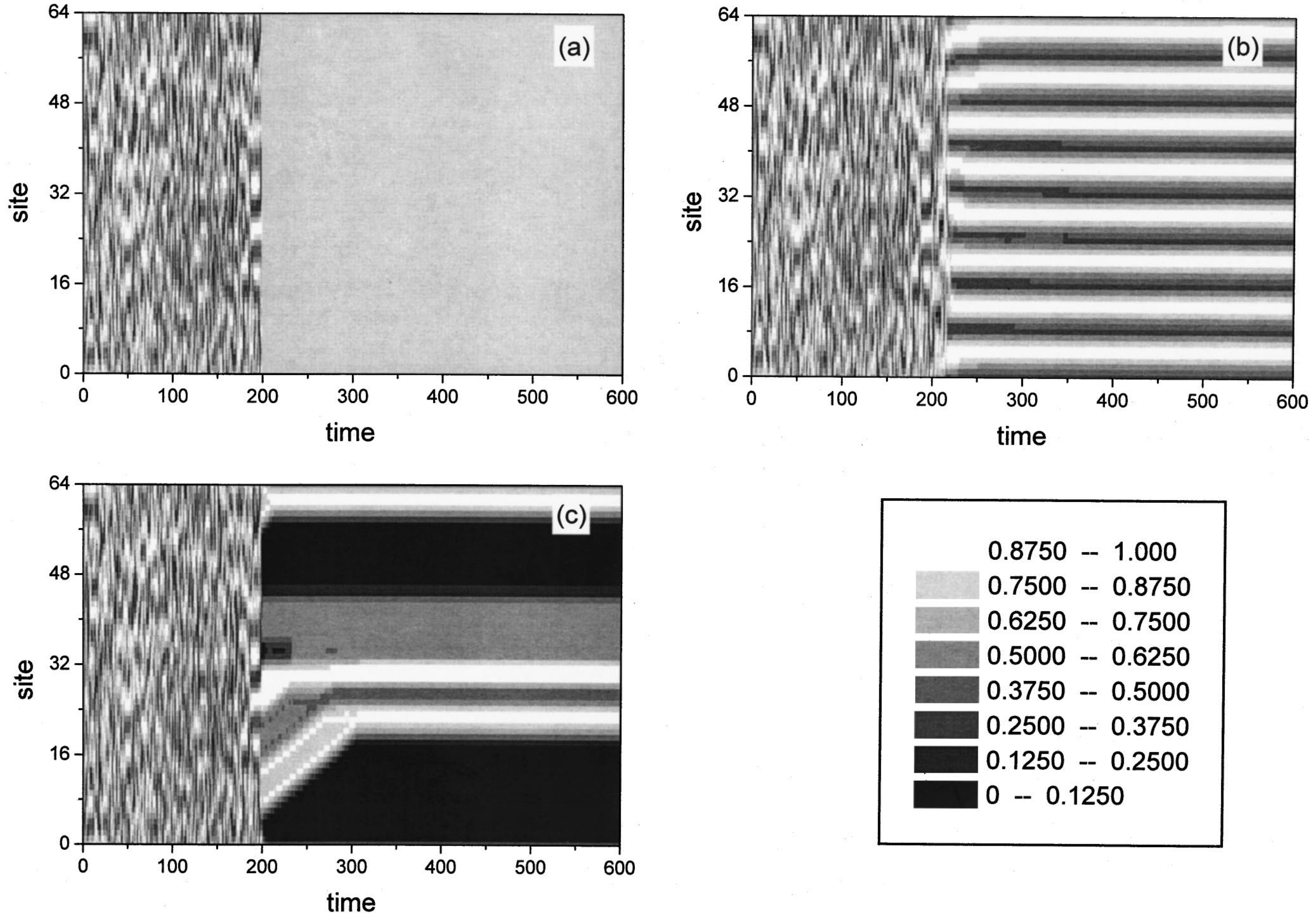

FIG. 2. Space-time diagram for CML (1) with $\varepsilon=0.8,\alpha=4$, $L=64$, and time$=n/8-1250$. Phase space compression starts at time 200. After control, (a) homogeneous steady state x^* is obtained with $x_{\max}=0.75$ and $x_{\min}=0$; (b) stable state of space period 8 is obtained with $x_{\max}=0.95$ and $x_{\min}=0.13$; (c) inhomogeneous stable state is obtained with $x_{\max}=1$ and $x_{\min}=0.29$.

and let $x_{\max}=x^*=0.75$; the turbulence of system (1) is controlled to the homogeneous steady state x^* very quickly when time is greater than 200. In Fig. 2(b), with $x_{\max}=0.95$ and $x_{\min}=0.13$, system (1) finally has a stable state of space-period-8, and the transient process under control is exhibited. In Fig. 2(c) an inhomogeneous stable state under control is shown with $x_{\max}=1$ (the upper boundary of V unchanged) and $x_{\min}=0.29$. Certainly, by selecting different compression parameters $x_{\max}$ and $x_{\min}$ to control STC in the homogeneous CML (1), one can also obtain other homogeneous/inhomogeneous stable states or much more complex patterns. In numerical simulations, we find that the chaotic attractor of CML (1) is sensitive to some control parameters. For instance, in Fig. 2(b) and Fig. 2(c), one will obtain different stable patterns if control is started at different times or other conditions are used. However, when $0\leqslant x_{\max}\leqslant 0.75$ and $x_{\min}=0$, STC in CML (1) is controlled to the uniform stable state $x_{n+1}(i)=x_c$, $i=2,\ldots,L-1$, and the control result x_c holds independent of the lattice size or other conditions. The curve of control result x_c versus $x_{\max}$ is the same as in Fig. 5 but in $0\leqslant x_{\max}\leqslant 0.75$, their functional relationship may be written

$$x_c=\alpha x_{\max}(1-x_{\max})\quad \text{for } x_{\min}=0. \tag{6}$$

Obviously, in a certain region Eq. (6) has the same form as the local dynamics, and its control result x_c is a function of only the system parameter α and the control parameters $x_{\max}$ and $x_{\min}$; it has nothing to do with the coupling strength ε. Equation (6) will be very useful for controlling STC in CMLs because the desired target state x_c can be obtained by appropriately choosing $x_{\max}$ from it, and its values include any numbers in the interval $[0,1]$. In this region sites $i=1,L$ are also controlled to stable states but different from x_c except for $x_c=x^*$. Only for $x_{\max}=0.75$ or $x_{\max}=0.25$ are all the sites of the lattice controlled to the homogeneous steady state x^*.

It is already well known that some nonlinear systems can exhibit chaotic motion for some parameter values and periodic/pseudoperiodic motion for other parameter values in space and time. The control method in this paper can transform the system from a turbulent state of STC into a periodic state in space and time when the system orbit is limited to the compressed phase space (and the system parameters are not changed). This is because the control works before the next evolution process and phase space compression compels the chaotic attractor to take only the values decided by the control when the orbit exceeds the subspace boundaries. These definite values decide the subsequent process accord-

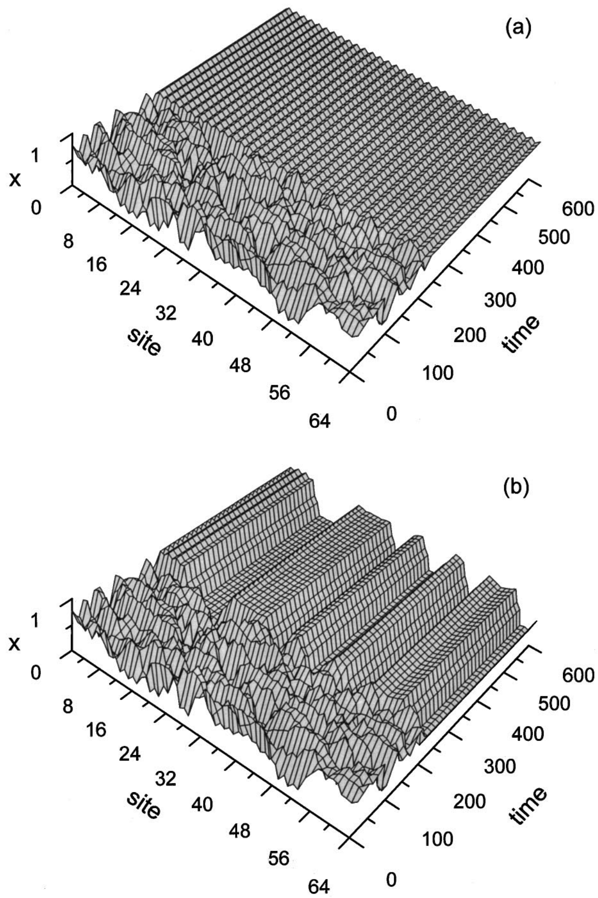

FIG. 3. Space-time-amplitude plot before and after control in heterogeneous CML (1) with $\alpha=3.9\pm0.1$, $\varepsilon=0.8\pm0.02$. Other conditions are the same as in Fig. 2. (a) STC is controlled to a stable state of space period 2 with $x_{\max}=0.75$ and $x_{\min}=0$. (b) STC is controlled to an inhomogeneous stable state with $x_{\max}=0.92$ and $x_{\min}=0.13$.

ing to the dynamical function, suppress the possible evolution orbits in the original phase space, and form the new orbit distributions in the system. In substance, phase space compression limits free contraction and expansion of the chaotic attractor in phase space, thus changing the system's dynamical character. Hence, by appropriately selecting different phase space compression parameters, one can control STC to different periodic or other states. In a word, the formation of the chaotic attractor of a nonlinear system needs both appropriate parameter values and a large enough phase space; changing either of these two conditions will vary the system dynamics.

2. Heterogeneous lattice

We change the coupling strength ε and nonlinear parameter α at the different lattice sites. Let $\varepsilon=0.8+(-1)^i\times0.02$, $\alpha=3.9+(-1)^i\times0.1$, $i=1,2,\ldots,64$; the other conditions are the same as in Fig. 2. Figure 3 shows the STC in heterogeneous CML (1) from time 0 to 200 and suppression of STC after control with all lattice sites chosen for the same phase space compression. The result in Fig. 3(a) is different

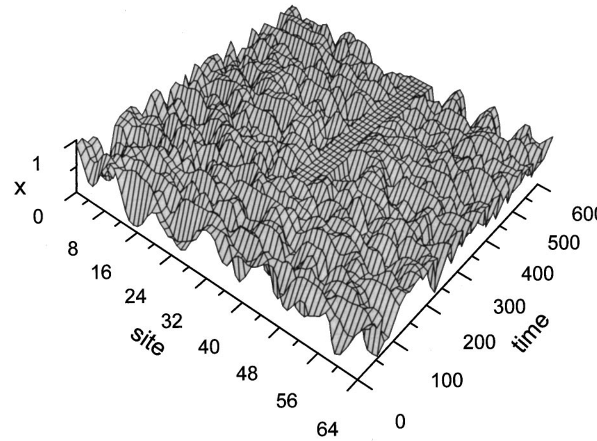

FIG. 4. Space-time-amplitude plot for local control. Sites $i_s=31,\ldots,38$ in the CML are selected for control with $x_{\max}=0.75$ and $x_{\min}=0$; other conditions are the same as in Fig. 2. Sites $i_p=33,\ldots,36$ are pinned and controlled to the uniform steady state x^*.

from that in Fig. 2(a) with $x_{\max}=0.75$ and $x_{\min}=0$. We find the heterogeneous system after control achieves a stable state of space period 2 that has two close values. As $0<x_{\max}\leq1-1/3.8$ and $x_{\min}=0$, space-period-2 continues to exist and each control result as a function of $x_{\max}$ obeys Eq. (6). In contrast to the homogeneous lattice, all sites of a heterogeneous lattice exhibit identical dynamics and obey the same control rule in the control parameter space although the parameters vary. Figure 3(b) shows that the turbulence is controlled to an inhomogeneous stable state.

B. Local control of STC

We use Eqs. (3) and (5) now, select the sites $i_s=31,\ldots,38$ in CML (1) exhibiting STC, and leave the rest undisturbed; spatially localized control is achieved after phase space compression is input. Figure 4 shows that the

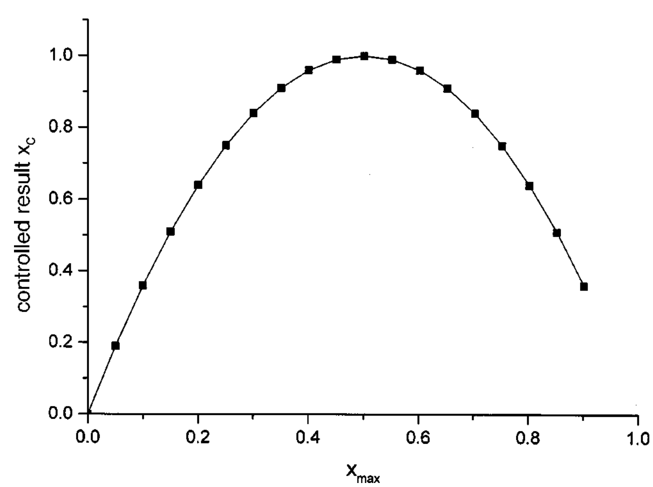

FIG. 5. Controlled result x_c versus $x_{\max}$ for local control with $x_{\min}=0$. Sites $i_s=31,\ldots,38$ in the CML are selected. The data points represent the numerical results for the sites that can be controlled to the uniform stable state x_c in a spatially localized region.

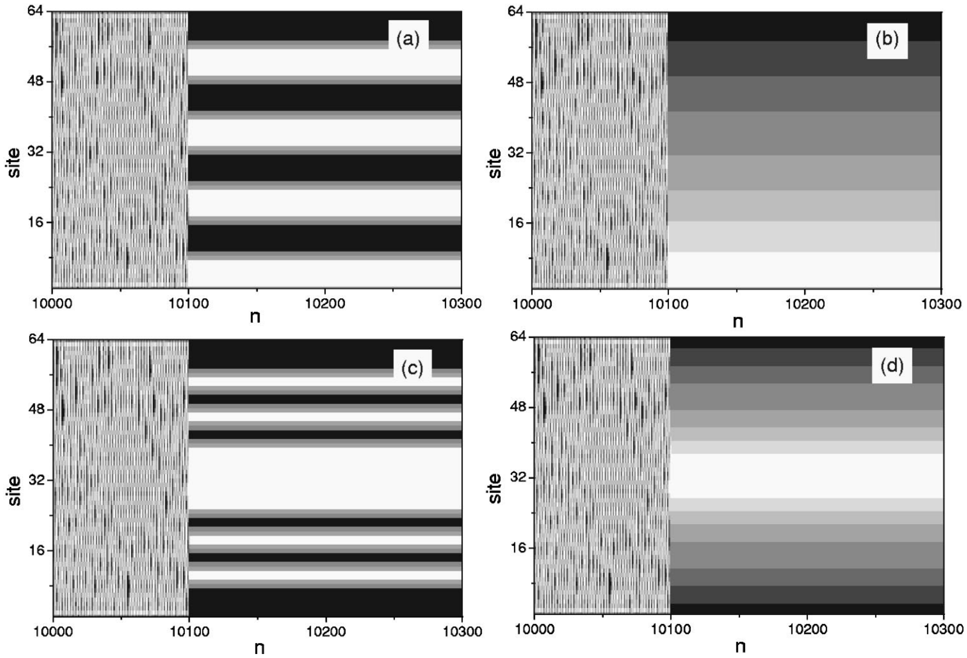

FIG. 6. Stable patterns obtained by different phase space compressions. Different x_{max} is selected in different control regions according to Eq. (6). Control begins at $n=10\,100$; every time step is plotted; other conditions are the same as in Fig. 2.

present method can be effectively used for controlling STC in spatially localized regions of a fully turbulent CML. The stabilized region, $i_p=33, \ldots ,36$, is smaller than the selected region. A different stable state in the locally controlled region can be obtained by choosing different phase space compression (x_{max} and x_{min}). Figure 5 shows the relationship of the control result x_c to x_{max} when $x_{min}=0$; it may be described by Eq. (6) at $0\leqslant x_{max}\leqslant 0.9$.

C. Global control of STC with different phase space compressions

We use different phase space compressions [select different $x_n(i)_{max}=x_{max}$ and $x_n(i)_{min}=x_{min}$ in different lattice regions] to control STC in CML (1) according to Eq. (6). The control results are shown in Fig. 6. In Fig. 6, phase space compression starts at $n=10\,100$ and every time step is plotted. x_{max} takes different values in different lattice regions when $\chi_{min}=0$; other conditions are the same as in Fig. 2. We obtain various desired stable patterns in the CML exhibiting STC. Certainly, by choosing different x_{max} according to Eq. (6) in different lattice regions, one can obtain other interesting patterns.

Finally, in numerical simulations we find that the states at the edges of the control regions are different from those in the control regions. We call this phenomenon the edge effect of controlling STC in CMLs by phase space compression.

III. CONCLUSIONS

We have shown that the method presented is effective for globally and locally controlling STC in CMLs via phase space compression. The fully developed turbulence in homogeneous or heterogeneous CML's is successfully controlled to homogeneous or inhomogeneous stable states by appropriately selecting the same global phase space compression. Spatially localized control of STC without disturbing the rest of the lattice can be effectively achieved by choosing appropriate local phase space compression. We find Eq. (6) whose form is the same as the local dynamics expression of a CML and the control results include any values in the phase space of the chaotic attractor. Using this control rule, we obtain various desired stable patterns by different phase space compressions [5]. Numerical simulations also show that Eq. (6) still holds for controlling STC in other CML models where the local function is the logistic map (for example, one-way coupled map lattice systems). The method avoids complex mathematical calculations in numerical simulations and may require only a multichannel threshold detector or a multichannel amplitude limiter in experiments. Controlling STC is a very difficult and significant task in real systems such as hydrodynamic systems, laser systems, chemical reactions, biological systems, and so on. We are sure the simple and effective method in this paper will have very important applications in practice.

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